The formats for linear (straight-line) equations can be written in simple variable expressions. The most common formats are slope-intercept form and point-slope form.

**Slope-Intercept form:**

\[ y = mx + b \]

- In the equation above, “m” is the slope of the line, or the **rise** (how far up or down the line moves) over the **run** (how far to the right the line moves).
- “b” is the y-intercept. It is the point at which the line crosses the y-axis.

**Example:** Find the equation for the line if you are given a slope of \( m = 3 \) and \( b = 0 \).

- We can find the equation for the line by plugging in the given values for “m” and “b” to the slope-intercept form.
- Our line can be written as \( y = 3x + 0 \), or just \( y = 3x \).
- With our equation \( y = 3x \), we can form a table to find the values of \( y \) by substituting in different values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 3x )</th>
<th>( y )</th>
<th>(( x, y )) coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = 3(0) )</td>
<td>0</td>
<td>(0,0)</td>
</tr>
<tr>
<td>1</td>
<td>( y = 3(1) )</td>
<td>3</td>
<td>(1,3)</td>
</tr>
<tr>
<td>2</td>
<td>( y = 3(2) )</td>
<td>6</td>
<td>(2,6)</td>
</tr>
</tbody>
</table>

**Point-Slope Form:**

\[ y - y_1 = m(x - x_1) \]

- In the equation above, the values for “\( y_1 \)” and “\( x_1 \)” are found in a given point that will lie on the line.
- “m” will be the slope of the line.

**Example:** Find the equation for the line if you are given the point (3,2) and a slope of \( m = -3 \).

- The value for \( x_1 \) would be 3, and \( y_1 \) would be 2. The slope will be \( m = -3 \).
- Plug the values in to the equation, and we find our line will be \( y - 2 = -3(x - 3) \).
  - We can isolate “\( y \)” on one side to find the slope-intercept form.
  - Our line can also be re-written as \( y = -3x + 11 \).
It is possible to find the equation of a line given just two points. This can be accomplished by finding the slope and substituting one of our points in to the point-slope formula.

**Example:** Find the equation for a line that contains both the points (1,8) and (7,-2).

- We will first need to find the slope of the line.
- The formula for the slope is equal to \( m = \frac{y_2 - y_1}{x_2 - x_1} \)
  - It does not matter which point we use for \((x_1,y_1)\) or \((x_2,y_2)\), but it needs to be consistent.
  - Substitute the values for \((x_1,y_1)\) with (1,8).
  - Substitute the values for \((x_2,y_2)\) with (7,-2).
  - \( m = \frac{(-2) - 8}{7 - 1} = -\frac{10}{6} = -\frac{5}{3} \)
  - The slope of our line is \(-\frac{5}{3}\)

- Now we can solve for the line using the point-slope formula with one of our points and the slope. It does not matter which point is used. Both points will find the same equation.
  - \( y - y_1 = m(x - x_1) \)
  - \( y - 8 = (-\frac{5}{3})(x - 1) \)
  - \( y - 8 = (-\frac{5}{3})x + \frac{5}{3} \)
  - The point-slope intercept form for the equation of the line that contains (1,8) and (7,-2) is \( y - 8 = (-\frac{5}{3})x + \frac{5}{3} \)
  - We can also change the line into slope-intercept form. This will solve for “b” and give the y-intercept for the line.
  - The slope-intercept form for the equation of the line that contains (1,8) and (7,-2) is \( y = (-\frac{5}{3})x + \frac{29}{3} \)
  - The y-intercept will be at the point \((0, \frac{29}{3})\)
Graphically, the x- and y-intercepts are the points where the equation will cross the x- or y-axis. Algebraically, the x-intercept is a point in the equation where the y-value is zero. The y-intercept is a point in the equation where the x-value is zero.

**Finding the x-intercept:**

To find the x-intercept, plug zero into the equation for y.

**Example:** Find the x-intercept for \( y = 4x + 2 \)

- Plug zero in for “\( y \)" in the equation
  - \( 0 = 4x + 2 \)
  - \( -4x = 2 \)
  - \( x = -\frac{1}{2} \)
- The x-intercept is \( (-\frac{1}{2}, 0) \).

**Finding the y-intercept:**

To find the y-intercept, plug zero into the equation for x.

**Example:** Find the y-intercept for \( y = 4x + 2 \)

- Plug zero in for “\( x \)” in the equation
  - \( y = 4(0) + 2 \)
  - \( y = 0 + 2 \)
  - \( y = 2 \)
- The y-intercept is \( (0,2) \).
Graphing a Line

We can graph lines either by using **option 1** the slope and y-intercept, **option 2** a table and plotting points or **option 3** the x- and y-intercepts.

**Example**: Graph the line $y = 3x + 2$

**Option 1.** Graph the line starting at the y-intercept and use the slope to plot coordinates.

- Our y-intercept is at $(0,2)$.
  - This can be found by plugging zero in to the equation, $y = 3(0) + 2$
- Plot $(0,2)$ on the y-axis.
- Using this point, graph the rise over the run.
- Since $m = 3$, the slope is $\frac{3}{1}$. The rise will be 3, and the run will be 1.
- The next point will be up 3 and right 1 from $(0,2)$. This will be at point $(1,5)$.
  - If slope is positive, the rise will move up. If slope is negative, the rise will move down.

**Option 2.** Create a table and substitute values in for $x$ and plot the $(x,y)$ coordinates.

- Substitute in values for $x$. For this graph, we start at $x = -3$. Solve the equation by substituting $-3$ in the equation, $y = 3(-3) + 2 = -7$.
- Select various values for $x$ and solve for the corresponding $y$ values.
- Plot the $(x,y)$ coordinates and connect the points to the graph line.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-7</td>
</tr>
<tr>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>
Option 3. Graph the line by using the x- and y-intercepts and finding a third point.

- Our y-intercept is at (0,2). This can be found by plugging zero for x in to the equation, \( y = 3(0) + 2 \)
- Plot (0,2) on the y-axis.
- Our x-intercept is at \((-\frac{2}{3}, 0)\). This can be found by plugging zero for y in to the equation, \( 0 = 3x + 2 \)
- Plot \((-\frac{2}{3}, 0)\) on the x-axis.
- Connect the points to graph the line.

Midpoint Formula:

The midpoint of 2 given points \((x_1,y_1)\) and \((x_2,y_2)\) is equal to \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\).

Example: Find the midpoint of \((4,6)\) and \((2,4)\)

- Plug in both given points in to the formula.
- \(\left(\frac{4+2}{2}, \frac{6+4}{2}\right) = \left(\frac{6}{2}, \frac{10}{2}\right) = (3,5)\) is the midpoint.

Slope of a parallel line:

The slope of a parallel line is the same slope as a given line with a different y-intercept. Parallel lines will never cross each other.

Example: \(y = 3x + 8\) and \(y = 3x - 4\) are parallel because they have the same slope of and different y-intercepts.

- The slopes in both equations are equal to 3. However, they have different y-intercepts.
Slope of a perpendicular line:

The slope of a perpendicular line is the opposite reciprocal slope of a given line. Perpendicular lines form a $90^\circ$ angle. This is also known as a right angle.

**Example:** $y = 3x - 4$ and $y = \left(\frac{-1}{3}\right)x + 2$ are perpendicular to each other because their slopes are opposite reciprocals.

- Opposite means a change in sign from the given slope (positive or negative).
- In this example, the first equation given has a slope of 3. The opposite of 3 is $-3$.
- Reciprocal means one divided by that number. The reciprocal of 3 is $\frac{1}{3}$.
- The opposite reciprocal of 3 is $-\frac{1}{3}$.
- A perpendicular line will have a slope equal to $-\frac{1}{3}$.

Parabolas:

$y = ax^2 + bx + c$

Parabolas are a two-dimensional, mirror-symmetrical curve. It is a quadratic function that looks U-shaped and opens either up or down depending on the function.

Parabolas can be graphed by creating a table and plotting points. However, understanding where vertex of the parabola is located will make graphing easier. Parabolas are typically written in the algebraic expression $y = ax^2 + bx + c$.

- “c” is the **y**-intercept.
- “a” determines whether if the parabola points up or down.
  - If “a” is negative, the parabola will point down.
  - If “a” is positive, the parabola will point up.
- To find where the turning point is, we need to determine where the minimum or maximum point is located. We can also determine the axis of symmetry. The axis of symmetry is a vertical line that passes through the turning point of a parabola.
- To find the axis of symmetry, use the formula $x = -\frac{b}{2a}$
- To find the vertex, plug the value of $x$ from the axis of symmetry and solve for $y$.
  - The vertex can be written as a point $(x, y)$. 
**Example**: Find the axis of symmetry for the parabola \( y = 4x^2 - 8x + 1 \).

- The axis of symmetry is equal to \( x = -\frac{b}{2a} \)
- Plug in the values from our parabola in to the formula to solve for the axis of symmetry.
- \( x = -\frac{(-8)}{2(4)} \)
- The axis of symmetry will be \( x = 1 \).

**Example**: Find the vertex for the parabola \( y = 4x^2 - 8x + 1 \).

- To find the vertex, we take the x-value from the axis of symmetry and plug it in to the equation of the parabola.
- The axis of symmetry was \( x = 1 \).
- Plug into \( y = 4(x)^2 - 8(x) + 1 \)
  - \( y = 4(1)^2 - 8(1) + 1 \)
  - \( y = -3 \)
- The vertex will be \((1, -3)\)

**Example**: Graph the parabola \( y = 4x^2 - 8x + 1 \).

- Now that we know the vertex is \((1, -3)\), we find 2 points to the left and right of the vertex.
- Create a chart

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
</tbody>
</table>
- Plot the points on the graph.
- Connect points using a symmetrical curve.